Welcome To . . .

Advanced Data Structures

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Many Thanks to Sartaj Sahni

What The Course Is About

- Study data structures for:
  - External sorting
  - Single and double ended priority queues
  - Dictionaries
  - Multidimensional search
  - Computational geometry
  - Image processing
  - Packet routing and classification
  - …

What The Course Is About

- Concerned with:
  - Worst-case complexity
  - Average complexity
  - Amortized complexity

Prerequisites

- ✓ C++
- ✓ Asymptotic Complexity
  - Big Oh, Theta, and Omega notations
- ✓ Undergraduate data structures
  - Stacks and Queues
  - Linked lists
  - Trees
  - Graphs

Kinds Of Complexity

- ✓ Worst-case complexity.
- ✓ Average complexity.
- Amortized complexity.

Quick Sort

- Sort n distinct numbers.
- Worst-case time is (say) $10n^2$ microseconds on some computer.
- This means that for every n, there is a sequence of n numbers for which quick sort will take $10n^2$ microseconds to complete.
- Also, there is no sequence of n numbers for which quick sort will take more than $10n^2$ microseconds to complete.

Quick Sort

- Average time is (say) $5n \log_2 n$ microseconds on some computer.
- Consider any n, say $n = 1000$.
- Add up the time taken to sort each of the $1000!$ possible 1000 element sequences.
- Divide by $1000!$.
- The result is $5000 \log_2 1000$.

Quick Sort

- What if we sort only 500 of these 1000! sequences?
- We can only conclude that the total time for these 500 sequences will be

  <= 500*(worst-case time)

  $= 500*(10n^2)$
- We cannot conclude that the time will be 500*(average time).

Task Sequence

- Suppose that a sequence of n tasks is performed.
- The worst-case cost of a task is c_{wc}.
- Let c_i be the (actual) cost of the i^{th} task in this sequence.
- So, $c_i <= c_{wc}$, $1 <= i <= n$.
- $n * c_{wc}$ is an upper bound on the cost of the sequence.
- $j * c_{wc}$ is an upper bound on the cost of the first j tasks.

Task Sequence

- Let c_{avg} be the average cost of a task in this sequence.
- So, $c_{avg} = \Sigma c_i/n$.
- $n * c_{avg}$ is the cost of the sequence.
- $j * c_{avg}$ is not an upper bound on the cost of the first j tasks.
- Usually, determining c_{avg} is quite hard.

Task Sequence

- At times, a better upper bound than $j * c_{wc}$ or $n * c_{wc}$ on sequence cost is obtained using amortized complexity.

Amortized Complexity

- The amortized complexity of a task is the amount you charge the task.
- The conventional way to bound the cost of doing a task n times is to use one of the expressions
  - n*(worst-case cost of task)
  - Σ(worst-case cost of task i)
- The amortized complexity way to bound the cost of doing a task n times is to use one of the expressions
  - n*(amortized cost of task)
  - Σ(amortized cost of task i)

Amortized Complexity

- The amortized complexity/cost of individual tasks in any task sequence must satisfy:

  Σ(actual cost of task i)

  <= Σ(amortized cost of task i)

- So, we can use

  Σ(amortized cost of task i)

  as a bound on the actual complexity of the task sequence.

Amortized Complexity

- The amortized complexity of a task may bear no direct relationship to the actual complexity of the task.

Amortized Complexity

- In worst-case complexity analysis, each task is charged an amount that is >= its cost.

  Σ(actual cost of task i)

  <= Σ(worst-case cost of task i)

- In amortized analysis, some tasks may be charged an amount that is < their cost.

  Σ(actual cost of task i)

  <= Σ(amortized cost of task i)

Arithmetic Statements

- Rewrite an arithmetic statement as a sequence of statements that do not use parentheses.
- a = x+((a+b)*c+d)+y;

  is equivalent to the sequence:

  z1 = a+b;

  z2 = z1*c+d;

  a = x+z2+y;

Arithmetic Statements

a = x+((a+b)*c+d)+y;

- The rewriting is done using a stack and a method processNextSymbol.
- create an empty stack;

```
for (int i = 1; i <= n; i++)
    // n is number of symbols in statement
    processNextSymbol();
```

Arithmetic Statements

a = x+((a+b)*c+d)+y;

- processNextSymbol extracts the next symbol from the input statement.
- Symbols other than) and ; are simply pushed on to the stack.

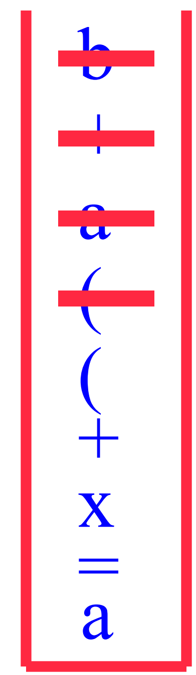
```
b
+
a
(
(
+
x
=
a
```

Arithmetic Statements

$$a = x+((a+b)*c+d)+y;$$

- If the next symbol is), symbols are popped from the stack up to and including the first (, an assignment statement is generated, and the left hand symbol is added to the stack.

$$z1 = a+b;$$



Arithmetic Statements

$$a = x+((a+b)*c+d)+y;$$

- If the next symbol is), symbols are popped from the stack up to and including the first (, an assignment statement is generated, and the left hand symbol is added to the stack.

$$z1 = a+b;$$
$$z2 = z1*c+d;$$

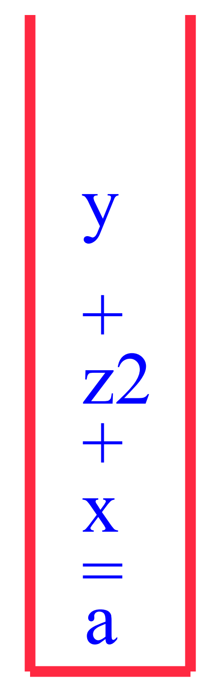
~~d~~
~~+~~
~~c~~
~~*~~
~~z1~~
~~(~~
+
x
=
a

Arithmetic Statements

a = x+((a+b)*c+d)+y;

- If the next symbol is), symbols are popped from the stack up to and including the first (, an assignment statement is generated, and the left hand symbol is added to the stack.

z1 = a+b;
z2 = z1*c+d;



Arithmetic Statements

$$a = x+((a+b)*c+d)+y;$$

- If the next symbol is ;, symbols are popped from the stack until the stack becomes empty. The final assignment statement
  $a = x+z2+y;$
  is generated.

$$z1 = a+b;$$
$$z2 = z1*c+d;$$

y
+
z2
+
x
=
a

Complexity Of processNextSymbol

a = x+((a+b)*c+d)+y;

- O(number of symbols that get popped from stack)
- O(i), where i is for loop index.

Overall Complexity (Conventional Analysis)

```
create an empty stack;
 for (int i = 1; i <= n; i++)
     // n is number of symbols in statement
     processNextSymbol();
```

- So, overall complexity is $O(\Sigma i) = O(n^2)$.
- Alternatively, $O(n*n) = O(n^2)$.
- Although correct, a more careful analysis permits us to conclude that the complexity is $O(n)$.

Ways To Determine Amortized Complexity

- Aggregate method.
- Accounting method.
- Potential function method.

Aggregate Method

- Somehow obtain a good upper bound on the actual cost of the n invocations of processNextSymbol()
- Divide this bound by n to get the amortized cost of one invocation of processNextSymbol()
- Easy to see that

  $\Sigma(\text{actual cost}) <= \Sigma(\text{amortized cost})$

Aggregate Method

- The actual cost of the n invocations of processNextSymbol()

  equals number of stack pop and push operations.
- The n invocations cause at most n symbols to be pushed on to the stack.
- This count includes the symbols for new variables, because each new variable is the result of a) being processed. Note that no)s get pushed on to the stack.

Aggregate Method

- The actual cost of the n invocations of processNextSymbol() is at most $2n$.
- So, using $2n/n = 2$ as the amortized cost of processNextSymbol() is OK, because this cost results in $\Sigma(\text{actual cost}) <= \Sigma(\text{amortized cost})$
- Since the amortized cost of processNextSymbol() is 2, the actual cost of all n invocations is at most $2n$.

Aggregate Method

- The aggregate method isn’t very useful, because to figure out the amortized cost we must first obtain a good bound on the aggregate cost of a sequence of invocations.
- Since our objective was to use amortized complexity to get a better bound on the cost of a sequence of invocations, if we can obtain this better bound through other techniques, we can omit dividing the bound by n to obtain the amortized cost.