Cross-Layers Cascade in Multiplex Networks

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ABSTRACT
The study of information cascade in multiplex networks where agents are connected by multiple linking types has received increasing interest. Comparing with the cascade in simplex networks, a noticeable characteristic of the cascade in multiplex networks is that information may be spread between multiple layers. Here, we focus on the cross-layers cascade which helps clarify two opposite opinions about the information cascade in multiplex networks: multiplexity can speed up or slow down information cascade. Two features of cross-layers cascade are proposed: the mapping relationship provides cross-layers paths; the vertical transfer coefficient quantifies the influences of agent varied in multiple layers. After generalizing the linear threshold model to multiplex networks, preconditions and reasons of seemingly paradoxical phenomena are discussed using three representative case studies and extensive simulations. It is found that the slow-down phenomenon emerges due to the obstruction of cross-layers cascade which connects the distributed shortest path in multiple layers. On the other hand, extra short paths or rapid spreading in one additional layer can respectively facilitate cascade process in existing networks. In conclusion, we think that the concept of cross-layers cascade may provide new insights into further study of information spreading in multiplex networks.

Categories and Subject Descriptors
I.2 [**Artificial Intelligence**]: Multiagent Systems; J.4 [**Computer Applications**]: Social and Behavioral Sciences

Keywords
Information Cascade, Multiplex Networks, Linear Threshold Model, Multiagent Systems.

1. INTRODUCTION
Cascade is an interesting phenomenon referring to a global diffusive process of a local effect which is initialized by one or small fraction of nodes in a network [1] [2]. In social life, the adoptions of an innovation and social norms [3] [4], the propagations of behaviors and opinions [5] [6], and catastrophic spreading of failures and epidemics [7-10] are well-known cascade phenomena. These diffusive processes are often studied by the linear threshold model [2] [11-13], in which a node becomes active if the influences of active neighbors exceed a predefined value. Traditional studies generally analyzed cascade process in simplex networks where linking types between nodes are identical. Recently, more and more studies have realized that real social networks contain multiple-layer structures mainly because social agents are connected by multiple linking types [14-18]. In multiplex networks, an agent can transfer information between layers besides spread information within each layer. For instance, a person can share topics in real life communication to online social networks or post his/her tweets (from Twitter) to Facebook. To the best of our knowledge, few studies have formally described the details of cross-layers cascade which helps to analyze the complicated effects of multiplexity on cascade processes.

One general opinion is that multiplexity can speed cascade process up [18-21]. References [18] and [19] reveal the dramatic effect of conjoining two entirely different networks on the velocity and scale of information cascade. In [20], a superdiffusive behavior is concluded which means that cascade process in multiplex networks is faster than the cascade in any disjointed layers. Meanwhile, to control cascade process in multiplex networks, adding or removing sparse layers in existing multiple layers is proved to be a feasible way [21]. In a word, it is generally accepted that multiplexity provides more feasible paths for information cascade. Indeed, people receive vast amount of information quickly from multiple channels every day; and many new fashions in online social networks have become hot topics in real life.

However, according to some real data, cascade processes always turn out slow as information spreads on the topologically inefficient path which means the propagation path is much longer than the shortest link between two randomly selected nodes in large scale networks [22-24]. It is known that the speeds of spreading information on distinct linking types are different [25] [26]. In real networks, information only flow easily on part of the edges, while cascade tends to be dampened in the rest part. For example, it is more natural to talk about a new washing machine with neighbors instead of members in physical training clubs if the advertisement is made on online social networks. Therefore, multiplexity may cause the slow-down phenomenon since information selectively propagates on networks and cannot be freely transferred from one layer to conjoining layers.

In this paper, we focus on the cross-layers cascade to understand and explain how the cascade process in multiplex networks is slowed down or speeded up. Two features of cross-layers cascade are proposed: the mapping relationship conjoins multiple layers and provides the transfer paths; the vertical transfer coefficient

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quantifies influences of node varied in multiple layers. The linear threshold model is generalized to multiplex networks, in which node becomes active if influences of active neighbors in any layer reach a predefined threshold [21]. Based on the cross-layers cascade, different cascade processes are scrutinized in three representative case studies and preconditions of seemingly paradoxical phenomena are discussed.

In multiplex networks, the shortest path between two randomly selected nodes is distributed in multiple layers. When the cross-layer cascade which connects the distributed shortest paths in two layers cannot be triggered, downstream nodes in the shortest path will be activated by the information spreading on the other topologically inefficient paths. As a result, slow-down phenomenon emerges comparing with the cascade process in simplex networks. Extra paths in additional layer which are shorter than existing propagation paths in other layers can facilitate the information spreading in multiplex networks. Large vertical transfer coefficient of additional layer causes the rapid cascade. What's more, rapid cascade in tiny-scale layer which contains only one hundredth of nodes in multiplex networks can induce global cascade since cross-layers cascades trigger many concurrent cascades in other layers. The leverage of tiny-scale layer on cascade process complements previous studies on the facilitation of multiplexity. Different reasons of slow-down and speed-up phenomena are also validated using extensive simulations. We hope that the introducing of cross-layers cascade can provide new insights into the study of information cascade in multiplex networks.

The rest of the paper is organized as follows. We outline the details of cascade across layers and cascade model in Section 2. In Section 3, we analyze the slow-down and speed-up phenomena in multiplex networks. Simulations results and analyses are presented in Section 4. In Section 5, we conclude our findings and point out the future outlook of our research.

2. MODEL OUTLINE

In this section, we first give the model of multiplex social networks and describe the features of cross-layers cascade. Then, we generalize the linear threshold model to multiplex networks.

2.1 Multiplex Networks

A network is always formulated as a graph $G = (V, E)$, in which V is the set of nodes and E is the set of edges linking nodes [26]. Agent and node are interchangeable concepts in the following. In this paper, we assume that V is the set of all agents in multiplex networks. Multiple and parallel graphs are usually used to represent multiplex networks [20] [21]. According to the categories of linking types $\{l_1, l_2, ... l_n\}$ [26], multiplex networks contain n layers which are denoted by $L_1, L_2 \dots L_n$, as shown in Figure 1.

It is worth noting that there are two methods to generate multiplex networks: one is "splitting" [15] [26]; the other is "combining" [18] [19] [21]. Splitting method means that nodes and edges of G are distributed in $L_1, L_2 \dots L_n$ as realizing the diverse linking types of simplex network. Combining method indicates that $L_1, L_2 \dots L_n$ are conjoint by the relationships between nodes in different layers if one agent takes part in interdependent cascade processes in different networks.

For simplicity, $L_1, L_2 \dots L_n$ also represent the set of agents in each layer. Meanwhile, a_i and b_i denote agents in L_1 and L_2. The letter i is the identification of agent. Agent has binary states: a_i^1 ($a_i^s = 1$) means a_i is active and a_i^0 ($a_i^s = 0$) means current state is inactive.Ωa_i^{L1} is the set of nodes linking to a_i in L_1 ($a_i \in L_1$, $\Omega a_i^{L1} \subseteq L_1$).

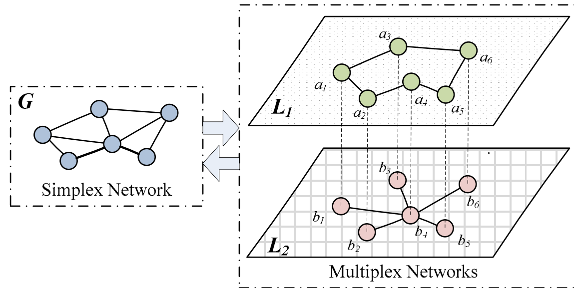


Figure 1 Illustration of Simplex and Multiplex Networks

2.2 Cross-Layers Cascade

2.2.1 Mapping Relationship

Mapping relationship indicates the dependence of states between agents. We first define the mapping relationship of agents in single layer. The symbol "→" is used to represent the correlation between agents. $\Omega a_i^{L1} \rightarrow a_i$ means the state of a_i depends on the neighbors of a_i. If $a_i \rightarrow a_j$ and $a_j \rightarrow a_i$, then $a_i \leftrightarrow a_j$. In single layer, mapping relationship likes the directed edge in graph theory [27]. Then, the mapping relationships of agents between layers are given.

Definition 2.1 b_i is injective to a_i, if $a_i \in L_1$, $\exists!\ b_i \in L_2$, such that $b_i \rightarrow a_i$ and b_i is not corresponding to a_i.

Definition 2.2 If $a_i \in L_1$, $\exists!\ b_i \in L_2$, and $b_j \in L_2$, $\exists!\ a_j \in L_1$, such that $a_i \leftrightarrow b_i$ and $b_j \leftrightarrow a_j$, then the mapping relationship between L_1 and L_2 is bijective.

Definition 2.3 The mapping relationship between L_1 and L_2 is multi-bijective, if $\forall\ a_i \in L_1$, $\forall\ b_j \in L_2$, and $\exists\ b_i \in L_2$, $\exists\ a_j \in L_1$, such that $\{b_i \dots\} \rightarrow a_i$, $\{a_j \dots\} \rightarrow b_j$.

The injective relation is unidirectional and provides a foundation for other mapping relationships. The multi-bijection is the main characteristic of interdependent infrastructure systems. For example, one power station can supply several nodes in the Internet communication network and several power stations may communicate through one or more communication nodes [7] [19]. Many online social networks, email networks and mobile communication networks can be considered as bijective multiplex social networks. In this paper, we focus on the cascade process under bijective relationship between layers. The symbol Φ is introduced: Φa_i^{L2} is the set of agents in L_2 that are $\Phi a_i^{L2} \subseteq L_2$ and $\Phi a_i^{L2} \rightarrow a_i$. For example, $\Phi a_1^{L2} = \{b_1\}$ in Figure 1.

Mapping relationship conjoins multiple layers and provides the paths for cross-layers cascades. Figure 2 shows that the cascade process in multiplex networks consists of cascade across layers and cascade on each layer. At first, a_1 and a_4 are set as active. Then, cross-layers cascades take place: b_1 and b_4 are activated. In next time step, a_2 is activated by a_1 in L_1 and b_6 becomes active due to b_4 in L_2. At last, b_2 and a_6 are activated, although b_2 is not linked to b_1 and a_6 is isolated from a_4. Therefore, without the consideration of cross-layers cascade, cascade on each layer is difficult to analyze.

In real multiplex social networks, it may take different times to transfer information between multiple layers. Time intervals of cross-layers cascades lead to asynchronous cascade processes [28]

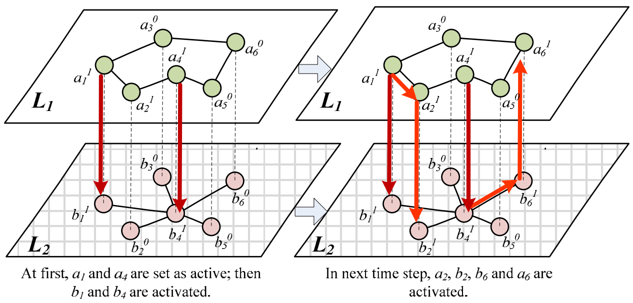


Figure 2 Illustration of Cascade Process in Multiplex Networks

in multiple layers: when b_i is activated by its neighbors in L_2, its mapping agent a_i in L_1 has been in active state for several time steps. In this paper, time interval of cross-layers cascade is assumed to be zero for the sake of simplicity. Only a simple case study is given to describe how different time intervals of cross-layers cascades further slow down cascade process in multiplex networks.

2.2.2 Vertical Transfer Coefficient

The vertical transfer coefficient proposed in this paper denotes the diverse influences of agent v in multiple layers. The symbol λ represents the vertical transfer coefficient and it is supposed that $\lambda \geq 0$. $\lambda b_i^{a_i}$ means the influence of b_i in L_2 transferred to a_i in L_1 if $a_i \in L_1$, $b_i \in L_2$ and $a_i^s \leftrightarrow b_i^s$. For example in Figure 2, the influence of b_4 received by b_6 is $\lambda a_4^{b_4} \times b_4^s$. For simplicity, if $\forall\ a_i \in L_1$ and $\Phi a_i^{L_2} = \{b_i\}$, $\lambda b_i^{a_i}$ is identical, then $\lambda L_2^{L_1}$ equals $\lambda b_i^{a_i}$ and represent the strength of L_2 mapping to L_1. Then, λ^{L_i} is named as the vertical transfer coefficient of L_i:

$$\lambda^{L_i} = \sum_{i \neq j}^{n} \lambda_{L_j}^{L_i}$$

Some related studies have defined similar parameters in multiple networks. In [29], the content-dependent parameters indicate different weighted links in each layer. Vertical transfer coefficient means the weight of the mapping relationship between layers.

2.3 Cascade Model

In this section, we generalize the linear threshold model to multiplex networks, in which node activates if the influences of neighbors in any layer is larger than the threshold [21]. In the classical linear threshold model, node v randomly chooses a threshold θ_v from the interval [0, 1]. Node v is linked with positive weight edges the sum of which is less than 1. Node v becomes active if the sum of weight edges linking to active neighbor exceeds the threshold θ_v [11-13]. Watt's threshold model [2] expands the linear threshold model and node v is activated if the fraction of active neighbors is larger than θ_v ignoring the weights of edges.

The principle of linear threshold model is that the activation of node or the decision of people to diffuse certain information needs reinforcements from multiple neighbors [5]. In real life or in the online social networks, it may be impossible for a person to estimate the number of all neighbors and calculate the fraction of active linking agents due to different social ties, familiarities, or communication intervals. It is more feasible to estimate the sum of influences of interactive neighbors. Therefore, node v in simplex network becomes active if

$$\tau_v \geq \theta_v .$$

τ_v is the sum of influences of neighbors of node v. The magnitude of agent's influence in simplex network is 1 unit. In multiplex networks, node v becomes active if

$$\max_{i=1,\dots,n}\left(\tau_v^{L_i}\right) \geq \theta_v .$$

$$\tau_v^{L_i} = \lambda^{L_i}\left(\sum \Omega_v^{\ L_i}\right).$$

The item $\sum \Omega v^{L_i}$ indicates the number of active neighbors of node v in L_i. With our generalized cascade model, agent can easily receive enough influences and widely spread its influence if the degree of agent is large. This assumption accords with reality. For example, if a person has many friends in Facebook or follows lots of people in Twitter, he/she will receive vast amount of information every day.

Cascade process highly depends on thresholds of nodes. Many studies have found the critical thresholds of global cascades in several kinds of networks [18] [20] [21], but the reliable formulas of threshold distributions according to different kinds of collective human behaviors still remain unknown [5]. In this paper, thresholds are assumed to follow normal and uniform distributions while the cascade condition and critical threshold are not the main aims.

3. ANALYSIS OF CASCADE PROCESS

In this section, cascade processes in multiplex networks are briefly analyzed in three case studies with the aid of cross-layers cascade. The reasons and preconditions of slow-down and speed-up phenomena are discussed.

Simplex network G is supposed to be a complete graph on four vertices which are denoted by v_1, v_2, v_3 and v_4. Thresholds of v_1 and v_2 are 1, while v_3 and v_4 become active if at least two neighbors are active. Node v_1 is initialized as active state. It takes three time steps to reach global cascade in G and the propagation paths are all shortest, as shown in Figure 3.

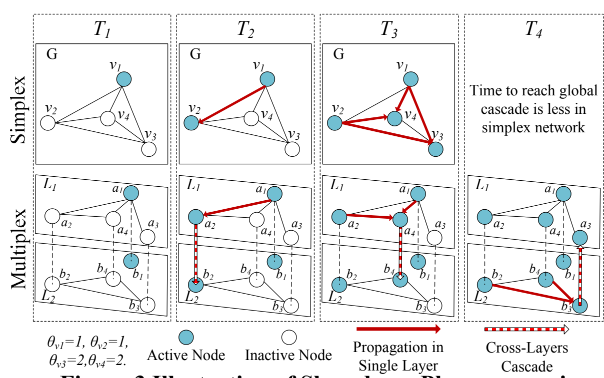


Figure 3 Illustration of Slow-down Phenomenon in Multiplex Networks

3.1 Case One

In the first case study, simplex network is split into two-layer multiplex networks [15] [26] and slow-down phenomenon in multiplex networks emerges. $\lambda L_2^{L_1}$ and $\lambda L_1^{L_2}$ are both 1 unit.

As shown in Figure 3, each layer contains part of edges in G and some agents cannot be activated in certain layer due to the isolation or lack of enough neighbors. For example, the activations of b_2 and b_4 depend on cross-layers cascades from a_2 and a_4 which can both become active following the shortest paths in G. Meanwhile, a_3 cannot be activated by neighbors in L_1 which only contains the first part of shortest path to active v_3 in G. The cross-layers cascade from a_3 to b_3 is obstructed since a_3 is surrounded by insufficient active nodes in L_1. One additional step is needed to activate b_3 (a_3, v_3) after the activation of b_4. However, v_3 and v_4 can be activated by v_1 and v_2 simultaneously in G.

Therefore, the shortest path in simplex network is distributed in multiple layers and the obstruction of cross-layers cascade is the main reason of slow-down phenomenon. If intermediate agent lacks sufficient neighbors in one layer which contains the first part of the shortest path, cross-layers cascade from the intermediate agent cannot take place and information spreading on the shortest path is blocked. Then, downstream nodes will be activated by information spreading on other longer paths which are topologically inefficient. As a result, slow-down phenomenon of cascade process in multiplex networks emerges comparing with the information spreading in simplex network.

With the consideration of time interval of cross-layers cascade, the global cascade will be further postponed. For example, if cascade from a_2 to b_2 falls behind cascade from a_4 to b_4, the activation of b_3 will be delayed.

It needs to be mentioned that traditional studies on social networks focused on the topology of agents' interactions without the consideration of the types of interactions. In other words, the topologically inefficient path found in empirical data may be the most efficient path in the framework of multiplex networks. Taking the mobile communication network as instance [22] [23], mobile phone users constitute the underlying simplex network which can be split into multiplex networks according to different linking types (multi-relation) [15] [22]. Therefore, whom a person calls must be related to the topic. Then, the edges of shortest path (whom a person knows) between two randomly selected nodes are distributed in multiple layers and many paths of cross-layers cascades are added. In social life, people consciously block cross-layers cascades just because people do not talk with every acquaintance about new fashions or share all new messages in online social networks.

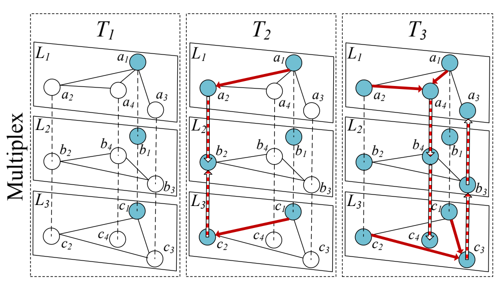


Figure 4 Illustration of Facilitation of Additional Layer in Multiplex Networks

3.2 Case Two

The second case study describes the facilitation of additional layer in multiplex networks, as shown in Figure 4. λ^{L1}, λ^{L2} and λ^{L3} are all 1 unit. Comparing with the multiplex networks in Figure 3, the structure of underlying simplex network remains the same, but L_3 provides an extra path to activate c_3 which is shorter than the existing path in L_2 and improves the speed of information cascade. By cross-layers cascade, mapping nodes in L_1 and L_2 are successively activated. However, the time to reach global cascade is still T_3.

It is worth noting that the acceleration of cascade process in multiplex networks emerges as compared with the case that the network contains disjoint multiple layers [18] [20]. Taking Figure 4 as instance, global cascade will not appear in any layer if the three layers are disjoint. By conjoining different networks, the increases of nodes' degrees and the structural changes in underlying simplex network are the probable reasons of speed-up phenomenon. Meanwhile, references [18] and [20] made no comparison between conjoint multiple layers and underlying simplex network.

In [21], Watt's threshold model is generalized to multiplex networks and node is activated if the proportion of active neighbors exceeds the threshold in any layer. The facilitation of multiplexity depends on the property of Watt's threshold model. Single layer may be unsusceptible to global cascade due to the constraint of network connectivity: sparse network lacks global connectivity; and node is always surrounded by insufficient proportion of active neighbors in dense layer. By coupling together or splitting a sparse layer from a dense network, most nodes easily become active in the sparse layer and influences of active nodes are widely spread due to the high connectivity in the dense layer.

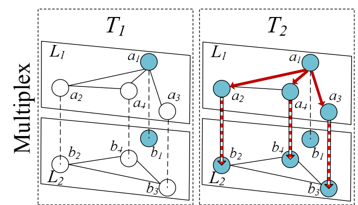


Figure 5 Illustration of Speed-up Phenomenon in Multiplex Networks

3.3 Case Three

The third case study shows the speed-up phenomenon in multiplex networks caused by rapid cascade in one layer (large transfer coefficient). $\lambda_{L_2}{}^{L_1}$ is set as 2 and $\lambda_{L_1}{}^{L_2}$ is still 1. In this case, the influence of a_1 equals two active agents. Then, agents in L_1 are activated by a_1 at T_2 and agents in L_2 become active because of cross-layers cascades simultaneously. The large value of transfer coefficient induces rapid cascade process in one layer and has positive effect on global cascade.

The third case study is simple but shows an interesting and common phenomenon in daily life. It is supposed that L_1 denotes Facebook or Twitter, L_2 is the word of mouth communication network for acquaintances [18], and a_1 represents a famous music star. Then, Figure 5 means that a_2, a_3 and a_4 are "fans" of a_1 but have no personal relationships with a_1 (the music star is isolated in L_2). If a_1 uploads a new song in online social network, a_2, a_3 and a_4 know it immediately without talking to each other in L_2. Indeed, new fashion spreads very fast and can become hot topic mostly due to the rapid cascade process in online social networks instead of the one in word of mouth communication.

On the other hand, if vertical transfer coefficient of certain layer is much larger, cascade processes in other layers may be inhibited. In Figure 5, no nodes are activated in L_2 as the active states are all vertically transferred from L_1. However, if a_3 is not the fan of a_1 in L_1, b_3 will know the new song from the conversations between b_2 and b_4.

4. SIMULATION

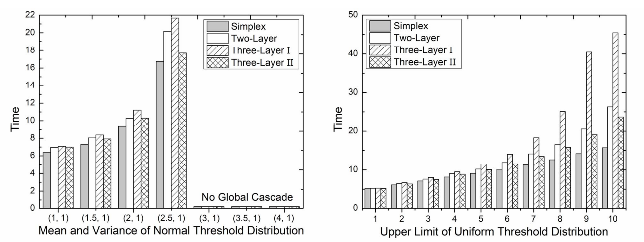
The cascade process in multiplex social networks has been simulated on a computer. According to many previous studies [18] [21] [22] [29], multiplex networks are constructed based on

Erdős–Rényi model [30] and small-world model [31]. Thresholds of nodes follow normal and uniform distributions.

Velocity and cascade size are two main parameters associated with the cascade process in networks. The velocity of cascade process in networks is evaluated by comparing the time to reach stationary state. The cascade size is measured by the average fraction of active nodes in stationary state. Each trial is performed with 100 replications. All the phenomena analyzed in three case studies are simulated.

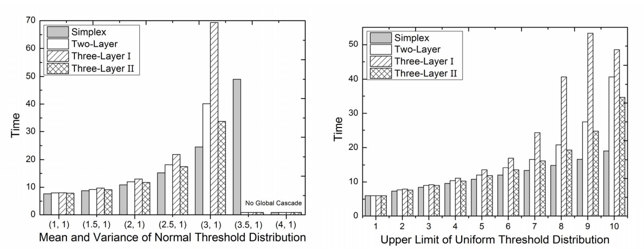
4.1 Which is Faster, Simplex or Multiplex?

The main object of this section is to compare the cascade processes in simplex and multiplex networks. Layer structures and threshold distributions are varied in different trials. We assume that the influence of node in simplex network equals 1 unit. The vertical transfer coefficients are constant to ensure that influences of node in multiple layers equal the one in simplex network.



(a) (b)

Figure 6 Time to Reach Stationary State with Erdős–Rényi Model



(a) (b)

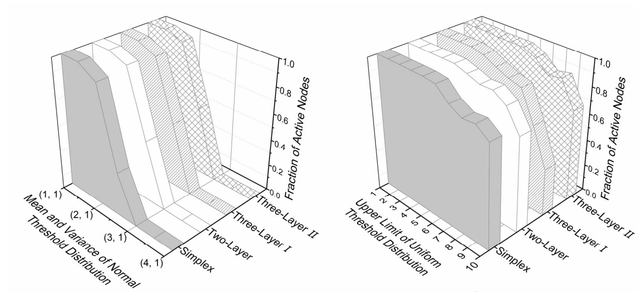
Figure 7 Time to Reach Stationary State with Small-World Model.

Based on the Erdős–Rényi model and small-world model, we first construct random simplex network and small-world simplex network both with 10000 nodes and average 20 neighbors. Meanwhile, in small-world simplex networks, the probability of interpolating between regular lattices is 0.1. Then, multiplex networks are generated: edges in simplex network are distributed to multiple layers according to given probabilities. There are three multiplex networks. Two-layer and three-layer (type I) multiplex networks indicate the multiplex networks in the first case study: each layer contains 1/2 and 1/3 of edges in simplex network and no edges are allocated twice. Three-layer (type II) multiplex networks represent the network model in the second case study: each layer contains half of edges in simplex network but edges can be allocated repeatedly. After that, one node is randomly initialized as active state. Then, information cascades are triggered in those simplex and multiplex networks independently. The results are shown from Figure 6 to Figure 9. The lower limit of uniform distribution is 0.

From Figure 6 and Figure 7, it can be clearly found that cascade processes are slowed down in multiplex networks because the shortest paths in simplex network are distributed in different layers. When threshold of node is low, times to reach stationary states in different networks are nearly the same because node can be easily activated in any layer and cross-layers cascade rarely takes place. However, as thresholds of nodes increase, the slow-down phenomenon in multiplex networks becomes more obvious. In this case, cross-layers cascade is more important because nodes with high thresholds become active only when enough neighbors are activated in other layers.

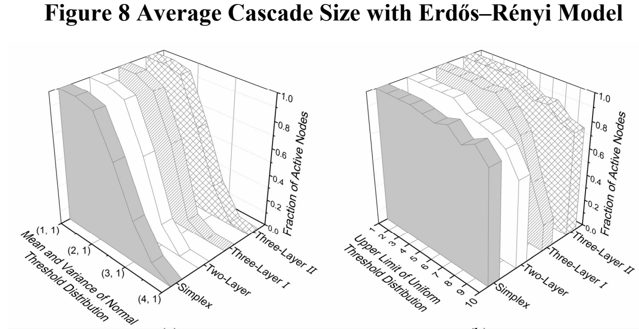
Meanwhile, information cascade in three-layer (type I) multiplex networks is slower than the one in two-layer multiplex networks. Therefore, the more layers are split from simplex network, the slower information spreading in multiplex networks will be if each edge is allocated only once. With our cascade model, if the degree of a node is large, node is easier to receive enough influences from neighbors to activate. On the contrary, if average number of neighbors in one layer is comparatively small, activations rely more heavily on the cascades from other layers. Comparing with two-layer multiplex networks, one additional layer with the same average number of neighbors in three-layer (type II) multiplex networks provides extra paths of information propagation. Thus, cascade processes in three-layer (type II) multiplex networks are faster than in two-layer multiplex networks but still slower than in simplex networks.

As shown in Figure 8 and 9, multiplexity also restricts the final scale of cascade process. Inhibition effect of multiplexity becomes more obvious when thresholds of nodes become larger. Meanwhile, if network models and threshold distributions are different, multiplexity shows diverse inhibition effects on cascade processes. The fraction of active nodes largely decreases in three-layer (type I) multiplex networks with uniform threshold distribution. However, with normal threshold distributions, cascade size in Erdős–Rényi network is less than in small-world network. Probable reason is that small-world network can provide more reinforcements from neighbors due to the high local clustering coefficient [5].



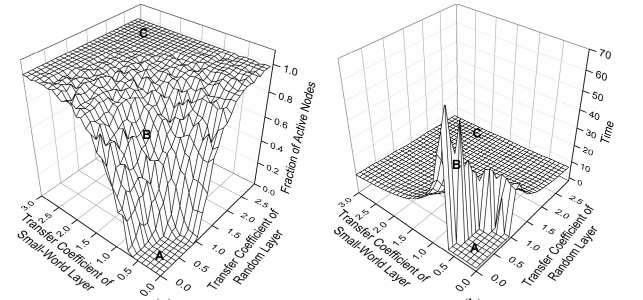
(a) (b)

Figure 8 Average Cascade Size with Erdős–Rényi Model



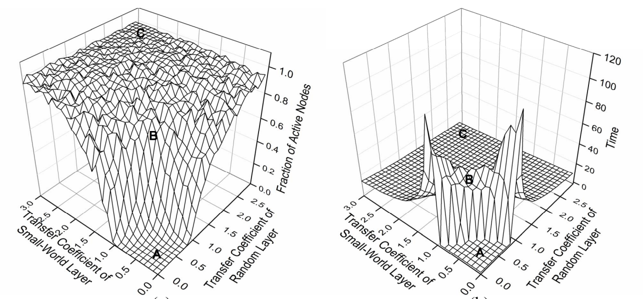
(a) (b)

Figure 9 Average Cascade size with Small-World Model



(a) (b)

Figure 10 Normal Threshold Distribution: N (2.5, 1)



(a) (b)

Figure 11 Uniform Threshold Distribution: U (0, 10)

4.2 Effect of Vertical Transfer Coefficient

The main object of this section is to show the effect of vertical transfer coefficient on cascade process as analyzed in the third case study. Layer structures and threshold distributions are constant in different trials. There are two layers in the simulated multiplex networks. To distinguish the two layers and avoid replications of simulations with same parameters, L_1 is small-world network with 10 neighbors and probability 0.1 of interpolating between regular lattices while L_2 is Erdős–Rényi network with average 10 neighbors. $\lambda_{L_2}^{L_1}$ and $\lambda_{L_1}^{L_2}$ are both varied from 0.1 to 3.0. Each layer contains 10000 nodes and $|V| = 10000$. Thresholds of nodes follow N(2.5, 1) normal distribution and U(0, 10) uniform distribution. The corresponding results are shown in Figure 10 and Figure 11.

The surfaces in Figure 10 and Figure 11 are divided into three areas, according to the areas of parameter spaces in which different final scales of cascade processes are found. A-area means seldom nodes are activated and propagation dies out quickly. Low transfer coefficients of two layers restrict the cascade processes. The shape of A-area in Figure 10 shows that the critical vertical transfer coefficient of random layer between A-area and B-area is larger than the one of small-world layer if vertical transfer coefficient of conjoint layer is set as 0. Comparing with Erdős–Rényi network, clustering coefficient of small-world network is much larger and one agent in small-world network can receive more reinforcements if one active neighbor activates other agents [5]. It means that Erdős–Rényi network with normal threshold distributions is not suitable for information spreading. However, the shape of A-area in Figure 11 indicates that cascade processes in random network and small-world network are similar with uniform threshold distributions.

As transfer coefficients of two layers increase, more nodes gradually become active and times to reach final prevalence achieve peaks rapidly and then decrease shapely. This parameter space is named as B-area which likes a fall in the graphs of fraction of active nodes and a ridge in the graphs of time to reach stationary state. Many cascade processes in reality belong to the B-area which is the transition region between global cascade and local popularity. In the B-area, multiplex networks contain many small groups of nodes which are susceptible to cascade process due to low thresholds but are separated by nodes with large thresholds. These separated groups of susceptible nodes mean that the sizes of final prevalence may be different if different nodes are initialized as active to trigger the cascade processes in the same multiplex networks.

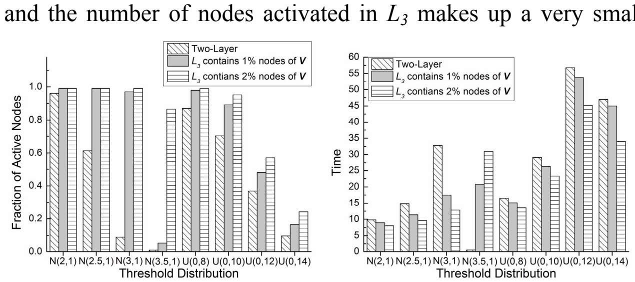
C-area means that global cascades emerge smoothly. As the third case study analyzed, the large value of vertical transfer coefficient leads to rapid cascade in one layer. Due to the cross-layers cascades, global cascade can take place in the multiplex networks even if the transfer coefficient of one layer is very small. Taking the online social network and word of mouth communication network as instance, new fashions become widely known mostly because of the rapid spreading in online social network.

4.3 Leverage of Tiny-Scale Layer

In this section, our aim is to present and discuss the leverage of tiny-scale layer on the global cascade in multiplex networks: largely increasing the fraction of active nodes in stationary state and reducing the time of cascade process.

Multiplex networks contain two or three layers and $|V| = 10000$. L_1 is small-world network with 10 neighbors and probability 0.1 of interpolating between regular lattices while L_2 is Erdős–Rényi network with average 10 neighbors. Each layer contains all nodes of V. 1% or 2% nodes are randomly selected from V and constitute L_3 according to Erdős–Rényi model with average 3 or 6 neighbors. Thus, the scale of L_3 is very small and connectivity is also sparse. Vertical transfer coefficients are constant. Influences of nodes in L_1 and L_2 are set as 1 unit. The vertical transfer coefficient of tiny-scale L_3 (λ^{L_3}) are 5 unit as we want to analyze the effects of quick propagation in tiny-scale layer on other layers. L_3 is only set as random layer because propagations in Erdős–Rényi and small-world networks are similar (quick global cascading) when transfer coefficients are large as shown in Section 4.2. Threshold distributions are varied. One node is set as active state to trigger cascade processes in multiplex networks.

As mentioned above, information spreads quickly in L_3 because the influence of node is large in L_3, while cascade processes in L_1 and L_2 are much slower. The corresponding results are shown in Figure 12. It can be found that the final sizes of cascade processes are raised from local popularity to global cascade by adding L_3 into multiplex networks if thresholds of nodes follow normal N (3, 1) and N (3.5, 1) distributions. As analyzed in the second case study, more layers added into multiplex networks can facilitate the cascade process more or less if additional layers can provide extra short paths for information cascade. However, L_3 is sparse and the number of nodes activated in L_3 makes up a very small



(a) (b)

Figure 12 Leverage of Tiny-Scale Layer on Cascade Processes

proportion of the final cascade size in multiplex networks. Global cascade takes place because separated but susceptible groups in L_1 and L_2 can be activated and conjoint. When nodes in L_3 are activated, relational mapping nodes in L_1 and L_2 are activated because of cross-layers cascades. Then, nodes in susceptible groups are activated and isolation caused by high threshold nodes gradually disappears. This facilitation effect is named as the leverage of tiny-scale layer on cascade processes in multiplex networks.

However, if thresholds of nodes follow uniform distributions, the facilitation effect of rapid cascade in L_3 is limited. When the upper limits of uniform threshold distributions are larger than average degrees of L_1 and L_2, many nodes cannot be activated even if all linking neighbors are active. Meanwhile, most nodes can be easily activated in two-layer multiplex networks if the upper limits of uniform threshold distributions are smaller than average degrees of L_1 and L_2. In spite of this, the percentage increment of fraction of active nodes by adding L_3 is much larger than the scale of L_3.

The leverage of tiny-scale layer is a complement to the previous studies on the facilitation of multiplexity. In [21], Brummitt et al. suggest that cascade process in multiplex networks can be controlled by adding or removing sparse layer, but the sparse layer contains most part of nodes in dense layer and the simulated multiplex networks. Our work suggests that tiny-scale layer which contains only one hundredth of nodes in multiplex networks is also important. The superdiffusive behavior concluded in [20] means that cascade process in multiplex networks is faster than in any disjoining layers. In our work, fraction of active nodes in L_3 reaches stationary state much faster than in the multiplex networks. The scale of L_3 is very small and information propagates quickly. Cascade processes in some susceptible groups in L_1 and L_2 will not take place until the information has been transferred from L_3. In real social life, human behaviors always fall behind the cascade processes in online social networks. In spite of the time interval of cross-layers cascade, only prevailing information which has activated a large fraction of nodes in online social networks can become the hot topics in word of mouth network or even induce other collective behaviors such as panic buying and protest movement. The leverage of tiny-scale layer is also different from the effect of hub nodes in networks [10]. Hub nodes have much larger amount of neighbors than other nodes in networks. Nodes in L_3 are randomly selected and have similar numbers of neighbors in the underlying simplex network.

5. DISCUSSION AND FUTURE WORK

In this paper, we focus on the role of cascade across layers in the information propagation in multiplex networks. Mapping relationship and vertical transfer coefficient are proposed to be the main features of cross-layers cascade: one conjoins multiple layers and provides the paths for information spreading between layers; the other one quantifies influences of one node varied in multiple layers. After giving the generalized linear threshold cascade model, we analyzed how multiplexity slows down or speeds up information cascades based on the cross-layers cascade.

The main reason of slow-down phenomenon of information spreading is the obstruction of cross-layers cascade which connects the distributed shortest path in multiple layers. When the information spreading on the first part of shortest path in one layer cannot be transferred to the next intermediate nodes in other layers, downstream nodes can only be activated by the cascade processes on other topologically inefficient paths. As a result, time to reach global cascade in multiplex networks is longer than in simplex networks. However, the topologically inefficient path reported in the research of empirical data may be the most efficient in the framework of multiplex networks since information selectively propagates on networks. On the other hand, information can spread in a particular part of social agents more pertinently with the consideration of diverse linking types (multi-relation). For example, Google+ allows users to arrange neighbor nodes and share information in different "Circles" according to different relationships. "Circles" restrict the velocity and range of spreading information but help to protect privacy of users and avoid the troubles due to wide dissemination of information with no restriction. In other words, cross-layers cascade can be carefully controlled by users with the aid of these subjectively created "Circles".

Extra short paths and rapid spreading in the additional layer can both facilitate cascade processes in multiplex networks comparing with disjointed layers. The effect of popular online social networks on information spreading is similar to the one of additional layers conjoining to traditional communication networks, since users can make friends and share information with strangers conveniently in online social networks. The leverage of tiny-scale layer on global cascade indicates the difficulties of predicting or controlling cascade process in multiplex networks. Due to the cascade across layers, nodes activated in tiny-scale layer can trigger concurrent cascade processes in the conjoining large-scale layer if there are many susceptible but separated groups in multiplex networks.

The issue of information cascade in multiplex social networks may provide a basis for further exploration in other multi-agent systems such as normative multi-agent systems [32] and trust systems [33] where the role of simplex network topologies has been widely investigated. Similar to the different speeds of information cascades in multiple layers, the rule of norm evolution or the time to reach convention in each layer of multiplex networks may also be different. Meanwhile, the trust path for the selection of trustworthy service analyzed in [33] is probably distributed in multiple layers and connected by many cross-layers paths.

In future work, we would like to make a more detailed description of the cross-layers cascade and apply it to real multiplex networks. The effects of time interval of cross-layers cascade, threshold distribution and layer structure on cascade process will also be further analyzed since present simulations show threshold distribution and layer structure can influence cascade process. What's more, formalized description of cascade process in multiplex networks depending on the cross-layers cascade is needed. The research of multiplex networks is attracting more and more attention, but the real field data of multiplex social networks is still rare. Main difficulties are how to judge social agents are conjoint in different networks and track the information spreading on and across networks together. We anticipate that the concept of cross-layers cascade can inspire further study of information spreading in multiplex networks.

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